Online Appendix

Does Openness Amplify Money-Financed Fiscal Stimulus? A Fiscal Theory Perspective

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Appendix A: Non-policy Block

This appendix presents the full specification of the non-policy block, which follows the standard New Keynesian small open economy framework of Gali and Monacelli (2005, 2016) and Okano and Eguchi (2024).

A.1 Households

The small open economy has a representative household with a continuum of members indexed by $j \in [0, 1]$.

The household utility function is

$$\sum_{t=0}^{\infty} \beta^t \left[\mathcal{U}\left(C_t, L_t, N_t; Z_t\right)\right], \tag{A.1}$$

where, $C_t \equiv \frac{1}{(1-\nu)^{1-\nu}\nu^\nu} C_{H,t}^{1-\nu} C_{F,t}^{\nu}$ denotes the consumption index, $C_{H,t} \equiv \left[\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ is the index of domestic goods consumption, $C_{F,t} \equiv \left[\int_0^1 C_{F,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ is the quantity of a composite foreign good consumed, $\epsilon > 0$ is the elasticity of substitution between goods, $\nu \in [0, 1]$ is the measure of openness, $N_t \equiv \int_0^1 N_t(j)\, dj$ is the hours of labor, Z_t is the exogenous preference shifter, $\beta \equiv \frac{1}{1+\rho} \in (0, 1)$ denotes the subjective discount factor, and ρ is the rate of time preference with $P_{H,t} \equiv \left[\int_0^1 P_{H,t}(j)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$ and $P_{F,t} \equiv \left[\int_0^1 P_{F,t}(j)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$.

Period utility is

$$\mathcal{U}\left(C_t, L_t, N_t; Z_t\right) \equiv \left[U\left(C_t, L_t\right) - V\left(N_t\right)\right] Z_t,$$

with $V(\cdot)$ increasing and convex and $U(\cdot)$ increasing and concave.

Optimal allocation of any given expenditure within each category of goods yields the following demand function:

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} C_{H,t} \quad ; \quad C_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\epsilon} C_{F,t}, \tag{A.2}$$

for all j. The optimal allocation of expenditures between domestic and foreign goods implies that

$$C_{H,t} = (1-\nu)\,\mathcal{S}_t^{\nu} C_t \quad ; \quad C_{F,t} = \nu \mathcal{S}_t^{-(1-\nu)} C_t. \tag{A.3}$$

The sequence of budget constraints is as follows:

$$\begin{aligned} P_t C_t + B_{H,t} + \mathcal{E}_t B_{H,t}^* + M_t &= B_{H,t-1}(1+i_{t-1}) + \mathcal{E}_t B_{H,t-1}^* \left(1 + I_{t-1}^*\right) + M_{t-1} + W_t N_t \\ &\quad - P_t TR_t + D_t, \end{aligned}$$

where $B_{H,t}$ denotes the nominal riskless one-period domestic government bond in domestic currency units held by domestic households, $B_{H,t}^*$ is the nominal riskless one-period foreign government bond in units of foreign currency held by domestic households, i_t is the domestic nominal interest rate, i_t^* is the foreign nominal interest rate, $\mathcal{E}_t$ is the nominal exchange rate (the price of foreign currency in units of the domestic currency), W_t is the nominal wage, and D_t is the nominal dividend. Note that $B_t = B_{H,t} + B_{F,t}$, where $B_{F,t}$ denotes nominal riskless one-period domestic government bonds in units of the domestic currency held by foreign households.

Dividing both sides of the previous expression by the CPI P_t yields:

$$\begin{aligned} C_t + \mathcal{B}_{H,t} + \mathcal{Q}_t \mathcal{B}_{H,t}^* + L_t &= \Pi_t^{-1} \mathcal{B}_{H,t-1}(1+i_{t-1}) + (\Pi_t^*)^{-1} \mathcal{Q}_t \mathcal{B}_{H,t-1}^* \left(1 + i_{t-1}^*\right) \\ &\quad + \Pi_t^{-1} L_{t-1} + \frac{W_t}{P_t} N_t - TR_t + \frac{D_t}{P_t}, \end{aligned} \tag{A.4}$$

where $\mathcal{B}_{H,t} \equiv \frac{B_{H,t}}{P_t}$ denotes real domestic government debt held by domestic households, $\mathcal{B}_t^* \equiv \frac{B_{H,t}^*}{P_t^*}$ is real foreign government debt held by domestic households, $\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}$ is the real exchange rate (the ratio of the CPI expressed in domestic currency), $\Pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$ denotes the (gross) foreign inflation, P_t^* is the foreign price index, and $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is (gross) CPI inflation.

Assuming complete international financial markets, the equilibrium price (in domestic currency units) of a riskless bond denominated in a foreign currency is $(1+i_t^*)^{-1} = Q_{t,t+1}\mathcal{E}_{t+1}$, where $Q_{t,t+1}$ denotes the price of a one-period discount bond paying off one domestic currency unit. We combine the previous pricing equation with the domestic bond-pricing equation, $(1+i_t)^{-1} = Q_{t,t+1}$ to obtain a version of the uncovered interest parity (UIP) condition:

$$Q_{t,t+1}\left[(1+i_t) - (1+i_t^*)\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)\right] = 0. \tag{A.5}$$

We define $\mathcal{A}_t \equiv \left[(1+i_{t-1})\,\mathcal{B}_{H,t-1} + \mathcal{Q}_{t-1}\mathcal{B}^*_{H,t-1}\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\left(1+i^*_{t-1}\right) + L_{t-1}\right]\Pi_t^{-1}$ as the representative household's real financial wealth at the beginning of period t. Therefore, we can rewrite Eq. (A.4).

$$C_t + \frac{1}{1+i_t}\mathcal{A}_{t+1}\Pi_{t+1} + L_t\left(1 - \frac{1}{1+i_t}\right) = \mathcal{A}_t + \frac{W_t}{P_t}N_t - TR_t + \frac{D_t}{P_t}, \tag{A.6}$$

where we assume the standard solvency constraint $\lim_{k\to\infty}\Lambda_{t,t+k}\mathcal{A}_{t+k} \geq 0$, ruling out the Ponzi scheme.

Households maximize Eq. (A.1), and subject to Eq. (A.6) and have the following optimality conditions:

$$U_{c,t} = \beta\,(1+i_t)\,\Pi_{t+1}^{-1}U_{c,t+1}\frac{Z_{t+1}}{Z_t}, \tag{A.7}$$

$$\frac{W_t}{P_t} = \frac{V_{n,t}}{U_{c,t}}, \tag{A.8}$$

$$h\left(\frac{L_t}{C_t}\right) = \frac{i_t}{1+i_t}, \tag{A.9}$$

with $h\left(\frac{L_t}{C_t}\right) \equiv \frac{U_{l,t}}{U_{c,t}}$. $h\left(\frac{L}{C}\right) \equiv \frac{U_l}{U_c}$ is a continuously decreasing function that satisfies $h\left(\bar{\chi}\right) = 0$ for some $0 < \bar{\chi} < \infty$. This guarantees that the demand for the real money balance is bounded as the interest rate approaches zero, with a satiation point attained at $L = \bar{\chi}C$. Eqs. (A.7), (A.8), and (A.9) are the consumption Euler equation and the intertemporal optimality condition that determine the labor supply under the assumption of a competitive labor market and money demand schedule, respectively. These optimality conditions require the TVC $\lim_{k\to\infty}\Lambda_{t,t+k}\mathcal{A}_{t+k} = 0$.

A.2 International Risk-sharing Condition

Assuming a complete financial market, a condition analogous to Eq. (A.4) must hold for a representative household in a foreign country. By combining this condition with Eq. (A.7) with UIP and the definition of the real exchange rate, we obtain the international risk-sharing condition

$$U_{c,t}^{-1} = \vartheta \left(U_{c,t}^{*}\right)^{-1} \mathcal{Q}_t \frac{Z_t}{Z_t^{*}}, \tag{A.10}$$

where $U_{c,t}^{*}$ denotes the counterpart of $U_{c,t}$ in the foreign country, Z_t^{*} denotes the foreign exogenous preference shifter, and ϑ is a constant that depends on initial conditions.

We assume LOOP; that is, $P_{F,t}(j) = \mathcal{E}_t P_{F,t}^{*}(j)$ for all j, where $P_{F,t}^{*}(j)$ denotes the price of foreign good j in units of foreign currency. By integrating all the goods, we obtain

$$P_{F,t} = \mathcal{E}_t P_{F,t}^{*}, \tag{A.11}$$

where $P_{F,t}^{*}$ denotes the foreign currency price of the foreign goods. Our treatment of the rest of the world as a (approximately) closed economy (with goods produced in a small open economy representing a negligible fraction of the world's consumption basket) implies that the foreign price index coincides with the foreign currency prices of foreign goods; that is, $P_t^{*} = P_{F,t}^{*}$.

By substituting the definition of CPI for the real exchange rate, we obtain

$$\mathcal{Q}_t = \mathcal{S}_t^{1-\nu}, \tag{A.12}$$

This implies that the assumption of complete markets at the international level leads to a simple relationship between consumption at home and abroad, and TOT. By substituting Eq.(A.12) into Eq. (A.10), we obtain

$$U_{c,t}^{-1} = \vartheta \left(U_{c,t}^{*}\right)^{-1} \mathcal{S}_t^{1-\nu} \frac{Z_t}{Z_t^{*}}. \tag{A.13}$$

A.3 Domestic Producers

A typical domestic firm produces a differential good using technology.

$$Y_t(j) = N_t(j)^{1-\alpha},$$

where $Y_t(j)$ is the output of generic good j and α denotes the index of decreasing returns to labor. The index for aggregate domestic output is $Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$. By integrating the previous expression, we obtain

$$N_t^{1-\alpha} = Y_t \left[\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\frac{\epsilon}{1-\alpha}} dj \right]^{1-\alpha}, \tag{A.14}$$

where $\int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\frac{\epsilon}{1-\alpha}} dj$ denotes price dispersion.

In each period, a subset of firms of measure $1-\theta$, with $\theta \in [0,1]$ being an index of price rigidities drawn randomly from the population, reoptimizes the price of their goods, subject to a sequence of isoelastic demand schedules for the latter. The remaining θ firms retain their prices. In other words, the firms are subject to Calvo pricing. Prices are set in the domestic currency, domestic and export markets share the same price, and the LOOP also applies to exports.

The first-order necessary condition (FONC) for domestic producers is

$$\sum_{k=0}^{\infty} \theta^k \left[\Lambda_{t,t+k} \left(\frac{1}{P_{t+k}} \right) Y_{t+k|t} \left(\tilde{P}_{H,t} - \mathcal{M} MC_{t+k|t}^n \right) \right] = 0, \tag{A.15}$$

where $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$ denotes the (desired) price markup, $Y_{t+k|t} \equiv Y_t \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon}$ is output in period $t+k$ for a firm that last reset its price in period t, $\tilde{P}_{H,t}$ is the price set in period t by firms reoptimizing their price in that period, $MC_{t+k|t}^n$ is the nominal marginal cost in period $t+k$ for a firm that last reset its price in period t, and $MC_t^n \equiv W_t \left(\frac{N_t^\alpha}{1-\alpha} \right)$ is the nominal marginal cost.

A.4 Demand for Exports and Global Shocks

The demand for exports of domestic goods j is

$$EX_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} EX_t, \tag{A.16}$$

where EX_t is the aggregate export index.

Following Gali and Monacelli (2016), aggregate exports are

$$EX_t = \nu \mathcal{S}_t Y_t^*, \tag{A.17}$$

where Y_t^* denotes (per-capita) world output.

A.5 The Market-clearing Condition

The market-clearing condition is

$$Y_t(j) = C_{H,t}(j) + EX_t(j) + G_t(j).$$

Plugging Eqs. (A.2), (A.3), (A.16), (A.17), and (A.10) into the previous expression, we obtain

$$Y_t = (1-\nu)\mathcal{S}_t^{\nu} C_t + \nu \mathcal{S}_t Y_t^* + G_t, \tag{A.18}$$

where we use the optimal allocation of the output $Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} Y_t$. We assume that $Y_t^* = C_t^*$, where C_t^* denotes the (per capita) world consumption.

Appendix B: Log-linearized Equilibrium Conditions

This appendix presents the full set of log-linearized equilibrium conditions. The notation follows Table B1.

B.1 Core Equilibrium Conditions

International Risk-sharing Condition

$$\hat{\xi}_t = -(1-\nu)s_t + \hat{\xi}_t^* - \zeta_t. \tag{B.1}$$

Market-clearing Condition

$$\hat{y}_t = \nu(2-\nu)s_t + (1-\nu)\hat{c}_t + \nu\hat{y}_t^* + \hat{g}_t. \tag{B.2}$$

Consumption Euler Equation

$$\hat{\xi}_t = \hat{\xi}_{t+1} + \hat{i}_t - \pi_{t+1} - \hat{\rho}_t. \tag{B.3}$$

Marginal Utility of Consumption

$$\hat{\xi}_t = -\sigma\hat{c}_t + \upsilon\hat{l}_t. \tag{B.4}$$

New Keynesian Phillips Curve (NKPC)

$$\pi_{H,t} = \beta\pi_{H,t+1} - \kappa\hat{\mu}_t, \tag{B.5}$$

where $\kappa \equiv \frac{(1-\theta\beta)(1-\theta)\Theta}{\theta}$ and $\Theta \equiv \frac{1-\alpha}{(1-\alpha)+\alpha\epsilon}$.

Price Markup Gap

$$\hat{\mu}_t = \hat{\xi}_t - \frac{\alpha+\varphi}{1-\alpha}\hat{y}_t - \nu s_t. \tag{B.6}$$

B.2 Money and Fiscal Variables

Money Demand Schedule

$$\hat{l}_t = \hat{c}_t - \eta\hat{i}_t. \tag{B.7}$$

Logarithmic First Differential of the Definition of the Real Money Balance

$$\hat{l}_{t-1} = \hat{l}_t + \pi_t - \Delta m_t. \tag{B.8}$$

Consolidated Government Budget Constraint

$$\hat{b}_t = (1+\rho)\,\hat{b}_{t-1} + (1+\rho)\,b\hat{i}_{t-1} - (1+\rho)\,b\pi_t + \hat{g}_t - \widehat{tr}_t - \chi\Delta m_t. \tag{B.9}$$

B.3 Price and Exchange Rate Definitions

Combination of the Logarithmic First Differential of the Definition of the CPI and TOT

$$\pi_t = \pi_{H,t} + \nu\left(s_t - s_{t-1}\right). \tag{B.10}$$

Definition of the Trade Balance

$$\widehat{nx}_t = \hat{y}_t - \nu s_t - \hat{c}_t - \hat{g}_t. \tag{B.11}$$

Definition of the TOT

$$s_t = e_t + p_t^* - p_{H,t}. \tag{B.12}$$

Definition of Domestic Inflation

$$\pi_{H,t} = p_{H,t} - p_{H,t-1}. \tag{B.13}$$

Definition of Import Inflation

$$\pi_{F,t} = p_{F,t} - p_{F,t-1}. \tag{B.14}$$

Combination of the (Logarithmic) Definition of the TOT with the (Logarithmic) Definition of Domestic and Import Inflation

$$\pi_{F,t} = s_t - s_{t-1} + \pi_{H,t}. \tag{B.15}$$

B.4 Parameter Definitions

The parameters are defined as follows:

- $\beta \equiv \frac{1}{1+\rho} \in (0,1)$: subjective discount factor
- ρ: rate of time preference
- $\mu \equiv \log\mathcal{M}$: constant (desired) price markup
- $\eta \equiv \frac{\epsilon_{lc}}{\rho}$, where $\epsilon_{lc} \equiv \frac{1}{\sigma_l+\upsilon}$: semi-elasticity of money demand
- $\sigma_l \equiv \frac{U_{ll}L}{U_l}$: elasticity of substitution between consumption and real balances
- $\chi \equiv \frac{L}{Y}$: steady-state inverse velocity of money
- $b \equiv \frac{\mathcal{B}}{Y}$: steady-state share of government debt to output
- $\varphi \equiv \frac{V_{nn}N}{V_n}$: curvature of labor disutility
- $\upsilon \equiv \frac{U_{cl}L}{U_c}$: separability of real balances
- $\sigma \equiv -\frac{U_{cc}C}{U_c}$: relative risk aversion

We assume $Z_t^* = 1$ and $Z_{t+1} = Z_t^{\varrho}$ with $\varrho = 0$. Therefore, $\hat{\rho}_t = \log Z_t = \zeta_t$ where $\hat{\rho}_t$ and ζ_t denote the demand and risk-sharing shocks, respectively.

Appendix C: Variable Definitions

Table C1 presents the definitions of all the logarithmic variables used in the analysis.

Table C1: Definition of Logarithmic Variables

Variable	Definition	Variable	Definition
$\hat{y}_t$	$\log\left(\frac{Y_t}{Y}\right)$	$\hat{b}_t$	$\frac{\mathcal{B}_t-\mathcal{B}}{Y}$
$\hat{c}_t$	$\log\left(\frac{C_t}{C}\right)$	$\widehat{tr}_t$	$\frac{TR_t-TR}{Y}$
$\hat{\rho}_t$	$-\log\left(\frac{Z_{t+1}}{Z_t}\right)$	$\widehat{nx}_t$	$\log\left[\left(\frac{NX_t}{P_{H,t}}\right)/Y\right]$
$p_{H,t}$	$\log P_{H,t}$	ζ_t	$-\log\left(\frac{Z_t^*}{Z_t}\right)$
$p_{F,t}$	$\log P_{F,t}$	$\hat{y}_t^*$	$\log\left(\frac{Y_t^*}{Y^*}\right)$
s_t	$\log \mathcal{S}_t$	ξ_t^*	$\log\left(\frac{U_{c,t}^*}{U_c^*}\right)$
$\hat{g}_t$	$\frac{G_t}{Y}$	p_t^*	$\log P_t^*$
ξ_t	$\log\left(\frac{U_{c,t}}{U_c}\right)$	e_t	$\log \mathcal{E}_t$
π_t	$\log \Pi_t$	$\pi_{H,t}$	$\log \Pi_{H,t}$
$\hat{i}_t$	$\log\left(\frac{1+i_t}{1+\rho}\right)$	$\pi_{F,t}$	$\log \Pi_{F,t}$
m_t	$\log M_t$	μ_t	$-\log MC_t$
$\hat{l}_t$	$\log\left(\frac{L_t}{L}\right)$	$\hat{\mu}_t$	$\mu_t - \mu$
$\widehat{sp}_t$	$\log\left(\frac{SP_t}{SP}\right)$		

Note: $MC_t \equiv \frac{MC_t^n}{P_{H,t}}$ denotes the real marginal cost. $\hat{\mu}_t$ is the markup gap.